Simulations of Reversible Navier-Stokes equation on logarithmic lattices

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We study the three-dimensional Reversible Navier-Stokes equations first introduced by Gallavotti[1] with conserved enstrophy, and later by Shukla[2] in which the energy is kept constant by adjusting the viscosity over time. We perform numerical simulations of these equations using a new framework called log-lattices, to reach extremely large resolutions at a moderate numerical cost. This technique allows us to explore regimes of parameters that were out of reach of the previous direct numerical simulations[2]. Using the non-dimensional forcing as a control parameter, and the square root of enstrophy as the order parameter, we confirm the existence of a second order phase transition well described by a mean field Landau theory. The log-lattices framework allows us to probe the impact of the resolution, highlighting an imperfect transition at small resolutions with exponents differing from the mean field predictions (Fig. 1(a)). Our findings are in qualitative agreement with predictions of a 1D non-linear diffusive model, the reversible Leith model of turbulence.

In addition, we analyze the Gallavotti conjecture[1], stating an equivalence of ensemble between NSE and its reversible counterpart for local observable. Under such conjecture, the macroscopic quantities of the irreversible system (NSE) could accurately be described by reversible equations (RNS) (Fig. 1(b)).

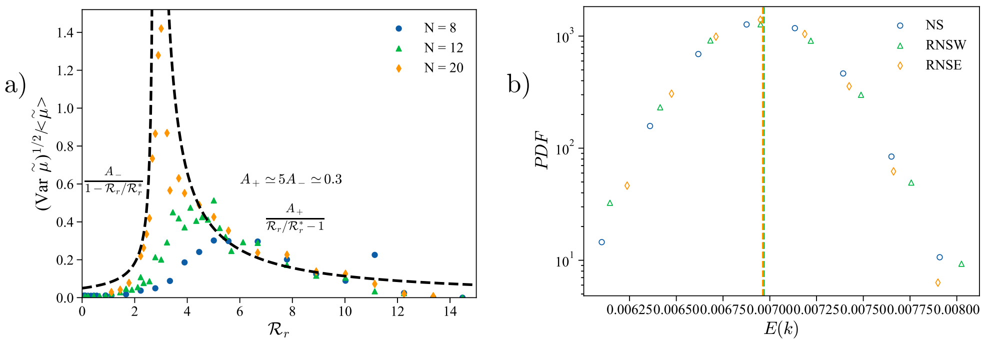

Figure 1: (a) Figure shows the variations of the square-root of the enstrophy, highlighting a second order phase transition, as a function of a dimensionless control parameter $\mathcal{R}_r = \frac{f_0}{E_0 k_f}$. f_0 being the forcing amplitude, E_0 the total, conserved, kinectic energy and k_f the injection scale. (b) Probability density functions of $E(k)$ at $k_s \approx 182.6$ for NSE and both conservation schemes (RNS), showing quasi-perfect agreement. In both figures, N corresponds to the number of modes.

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[1] Gallavotti, *Physics Letters A*. **223**, 91 (1996).

[2] Shukla et al., *Physics Review E* **100**, 043104 (2019).